

### Hohenberg-Kohn theorems & Effective Kohn-Sham Equation

- overcome from a single shoot both the *exchange and correlation* terms to the **total electronic energy** (THAT FUNCTIONAL the title of theory is referring at)
- **Observable Quantum Chemistry** (as based on electronic DENSITY; at its turn it can be experimentally expressed, e.g. X-Ray studies, etc.)
- **The Nobel Prize in Chemistry 1998 (Walter Kohn, John Pople)/Kohn Nobel Lecture:**

<https://www.nobelprize.org/prizes/chemistry/1998/kohn/lecture/>

$$N[\rho] = \int \rho(\mathbf{r}) d\mathbf{r} = \sum_i n_i$$

$$\rho(\mathbf{r}) = N \int \Psi^*(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_N) \Psi(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_N) d\mathbf{r}_2 \dots d\mathbf{r}_N$$

Electrons

(N-1) integrals

*V*-representability... "insurance for OBSERVABILITY"

$$\rho(\mathbf{r}) \geq 0, \forall \mathbf{r} \in \mathfrak{R} \quad \int_{\mathfrak{R}} |\nabla \rho(\mathbf{r})|^{1/2} d\mathbf{r} < \infty$$

$$\bar{\rho}(\mathbf{r}) = N \varphi^*(\mathbf{r}) \varphi(\mathbf{r}) \quad \text{"N-orbital uni-occupancy"}$$

---> **Effective uni-electronic chem-reactivity**

$$\left[ -\frac{1}{2} \nabla^2 + V_{\text{eff}} \right] \varphi(\mathbf{r}) = \mu \varphi(\mathbf{r}) \quad V_{\text{eff}}(\mathbf{r}) = V(\mathbf{r}) + \int \frac{\rho(\mathbf{r}_2)}{|\mathbf{r} - \mathbf{r}_2|} d\mathbf{r}_2 + V_{\text{xc}}(\mathbf{r})$$

$$E[\rho] = F_{HK}[\rho] + C_A[\rho]$$

$$F_{HK}[\rho] = T[\rho] + V_{ee}[\rho] + \int \rho(\mathbf{r}) V(\mathbf{r}) d\mathbf{r}$$

$$E[\bar{\rho}] \geq E[\rho] \Leftrightarrow \delta E[\rho] = 0$$

$$\delta \{E[\rho] - \mu N[\rho]\} = 0$$

Approx.

"Comput-Chem"

$$\bar{\rho}(\mathbf{r}) = \sum_i^n n_i |\varphi_i(\mathbf{r})|^2$$

$0 \leq n_i \leq 1$  vs.  $0 \leq n_i \leq 2$

$\sum_i n_i \sim N$  vs.  $\sum_i n_i = N$

$$\mu = -\chi = \left( \frac{\delta E[\rho]}{\delta \rho} \right)_{\rho=\rho(V)}$$

"Chem-Reactivity"